EE613: Estimation Theory
Problem Set 12

1. Consider the quadratic estimator

$$\hat{\theta} = ax^2[0] + bx[0] + c$$

of a scalar parameter θ based on the single data sample $x[0]$. Find the coefficients a, b, c that minimize the Bayesian MSE. If $x[0] \sim \mathcal{U}[-\frac{1}{2}, \frac{1}{2}]$, find the LMMSE estimator and the quadratic MMSE estimator if $\theta = cos2\pi x[0]$. Also, compare the minimum MSEs.

2. We observe the data $x[n] = s[n] + w[n]$ for $n = 0, 1, \cdots, N-1$, where $s[n]$ and $w[n]$ are zero mean WSS random processes which are uncorrelated with each other. The ACFs are

$$\begin{aligned} r_{ss}[k] &= \sigma_s^2 \delta[k] \\ r_{ww}[k] &= \sigma^2 \delta[k] \end{aligned} \quad (1)$$

Determine the LMMSE estimator of $\mathbf{s} = [s[0]s[1]\cdots s[N-1]]^T$ based on $\mathbf{x} = [x[0]x[1]\cdots x[N-1]]^T$ and the corresponding minimum MSE matrix.

3. In this problem we examine the interpolation of a data sample. We assume that the data set $\{x[n-M], \cdots, x[n-1], x[n+1], \cdots, x[n+M]\}$ is available and that we wish to estimate or interpolate $x[n]$. The data and $x[n]$ are assumed to be a realization of a zero mean WSS random process. Let the LMMSE estimator of $x[n]$ be

$$\hat{x}[n] = \sum_{k=-M, k\neq 0}^{M} a_k x[n-k]$$

Find the set of linear equations to be solved for the weighting coefficients by using the orthogonality principle. Next, prove that $a_{-k} = a_k$ and explain why this must be true. See also [Kay 1983][1] for a further discussion of interpolation.

4. Consider an AR(N) process

$$x[n] = -\sum_{k=1}^{N} a[k]x[n-k] + u[n]$$

where $u[n]$ is white noise with variance σ_u^2. Prove that the optimal one-step linear predictor of $x[n]$ is

$$\hat{x}[n] = -\sum_{k=1}^{N} a[k]x[n-k]$$

Also, find the minimum MSE.

[1] Kay, S., "Some Results in Linear Interpolation Theory", IEEE Trans. Acoustics, Speech and Signal Processing, vol. 31, pp. 746-749, June 1983