EE611 Solutions to Problem Set 4

1. The equivalent discrete-time filter model for the sampled matched filter output is (assuming a is real)

$$y_k = aI_{k-1} + (1 + a^2)I_k + aI_{k+1} + \nu_k,$$

where $\{\nu_k\}$ is zero-mean Gaussian with auto-correlation function

$$E[\nu_k \nu_{k+n}] = \begin{cases} \sigma^2 a & n = \pm 1 \\ \sigma^2(1 + a^2) & n = 0 \end{cases},$$

where σ^2 is the power spectral density of the channel AWGN.

2. Consider the channel model $v_k = I_k + 0.5I_{k-1} + \eta_k$.

(a) The possible values for $I_k + 0.5I_{k-1}$ are 1.5, 0.5, -0.5, and -1.5.

(b) The possible values for $I_k + 0.5I_{k-1}$ are (1.5, 1.5), (1.5, 0.5), (0.5, 1.5), (0.5, 0.5), (1.5, $-$ 0.5), (1.5, $-$ 1.5), (0.5, $-$ 0.5), (0.5, $-$ 1.5), ($-$0.5, 1.5), ($-$0.5, 0.5), ($-$1.5, 1.5), ($-$1.5, 0.5), ($-$0.5, $-$ 0.5), ($-$0.5, $-$ 1.5), ($-$1.5, $-$ 0.5), and ($-$1.5, $-$ 1.5).

The received signal constellations in the absence of noise are shown in Figure 1.

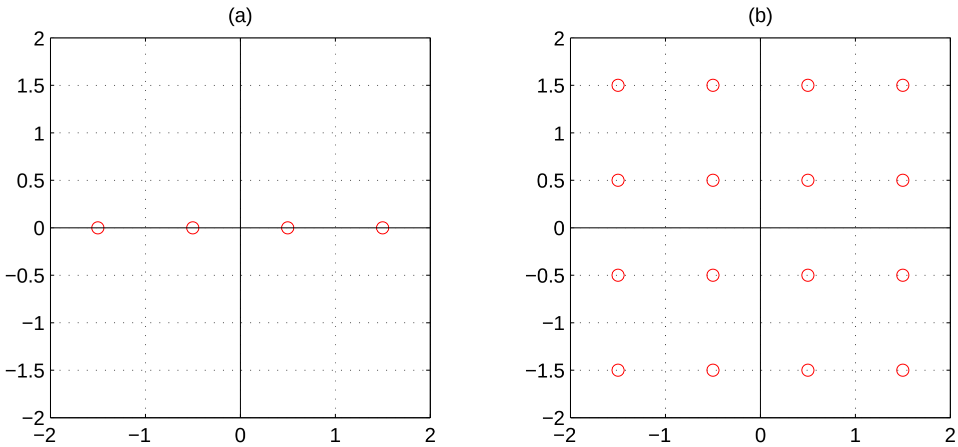

Figure 1:

For the channel model $v_k = I_k + I_{k-1} + \eta_k$, we have the following answers:

(a) The possible values for $I_k + I_{k-1}$ are 2, 0, and -2.

(b) The possible values for $I_k + I_{k-1}$ are (2, 2), (2, 0), (2, $-$ 2), (0, 2), (0, 0), (0, $-$ 2), ($-$2, 2), ($-$2, 0), and ($-$2, $-$ 2).

The received signal constellations in the absence of noise are shown in Figure 2.

3. (a) The state at time k S_k is $\{I_{k-1}\ I_{k-2}\}$. Since binary signaling is used, the number of possible states is 4. The trellis diagram is shown in Figure 3.

(b) Let $A_k = \{I_{k-1}\ I_{k-2}\}$ and $B_k = \{J_{k-1}\ J_{k-2}\}$. We know that $A_l = B_l$. Therefore, we have $I_{l-1} = J_{l-1}$ and $I_{l-2} = J_{l-2}$. Since $A_{l+1} \neq B_{l+1}$, we have $I_l \neq J_l$.

$$A_{l+2} = \{I_{l+1}\ I_l\} \quad \text{and} \quad B_{l+2} = \{J_{l+1}\ J_l\}\,.$$

Since $I_l \neq J_l$, we have $A_{l+2} \neq B_{l+2}$.

(c) From part (b), we can say that the minimum length of an error event is greater than or equal to 3. We can get an error event of length 3 if $I_k = J_k$ for $k = l-1,\ l+1,\ l+2$ and $I_l \neq J_l$. Therefore, the minimum length of an error event is 3.

(d) The error event is shown in Figure 4. The outputs corresponding to each branch of each path are also shown. The probability that the metric for path B is greater than the metric for path A is equal to

$$Q\left(\frac{d}{2\sigma}\right),$$

where σ^2 is the variance of the η_k, and

$$d = \sqrt{(1.75+0.25)^2 + (1.75-0.75)^2 + (1.75-1.25)^2} = 2.2913.$$

(e) Using arguments similar to part (b), the minimum possible length for an error event can be shown to be $L+1$.

4. (a) The 3-tap equalizer with coefficients c_{-1}, c_0, and c_1 can be obtained by solving the following equation

$$\begin{bmatrix} 1.25 & 0.5 & 0 \\ 0.5 & 1.25 & 0.5 \\ 0 & 0.5 & 1.25 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Therefore, we have $c_{-1} = -0.4706$, $c_0 = 1.1765$, and $c_1 = -0.4706$.

(b) $q_2 = q_{-2} = 0.5c_1 = -0.2353$.

(c) The 5-tap equalizer with coefficients c_{-2}, c_{-1}, c_0, c_1, and c_2 can be obtained by solving the following equation

$$\begin{bmatrix} 1.25 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1.25 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1.25 & 0.5 & 0 \\ 0 & 0 & 0.5 & 1.25 & 0.5 \\ 0 & 0 & 0 & 0.5 & 1.25 \end{bmatrix} \begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Therefore, we have $c_{-2} = 0.2462$, $c_{-1} = -0.6154$, $c_0 = 1.2923$, $c_1 = -0.6154$, and $c_2 = 0.2462$.

5. (a) The 3-tap MMSE equalizer with coefficients c_{-1}, c_0, and c_1 can be obtained by solving the following equation

$$\begin{bmatrix} 2.0625+0.125 & 1.25+0.05 & 0.25 \\ 1.25+0.05 & 2.0625+0.125 & 1.25+0.05 \\ 0.25 & 1.25+0.05 & 2.0625+0.125 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.25 \\ 0.5 \end{bmatrix}.$$

Therefore, we have $c_{-1} = -0.2722$, $c_0 = 0.8949$, and $c_1 = -0.2722$.

The 5-tap MMSE equalizer with coefficients c_{-2}, c_{-1}, c_0, c_1, and c_2 can be obtained by solving the following equation

$$\begin{bmatrix} 2.1875 & 1.3 & 0.25 & 0 & 0 \\ 1.3 & 2.1875 & 1.3 & 0.25 & 0 \\ 0.25 & 1.3 & 2.1875 & 1.3 & 0.25 \\ 0 & 0.25 & 1.3 & 2.1875 & 1.3 \\ 0 & 0 & 0.25 & 1.3 & 2.1875 \end{bmatrix} \begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 1.25 \\ 0.5 \\ 0 \end{bmatrix}.$$

Therefore, we have $c_{-2} = 0.1296$, $c_{-1} = -0.4178$, $c_0 = 1.0384$, $c_1 = -0.4178$, and $c_2 = 0.1296$.

(b) For the 3-tap MMSE equalizer, we have $q_{-2} = -0.13610$, $q_{-1} = 0.10720$, $q_0 = 0.84643$, $q_1 = 0.10720$, and $q_2 = -0.13610$.

For the 5-tap MMSE equalizer, we have $q_3 = q_{-3} = 0.064800$, $q_2 = q_{-2} = -0.046900$, $q_1 = q_{-1} = 0.061750$, and $q_0 = 0.880200$.

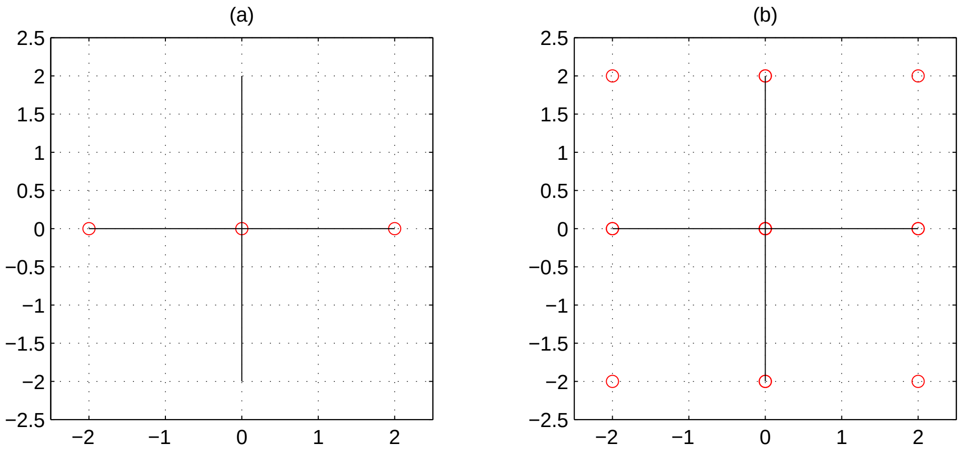

Figure 2:

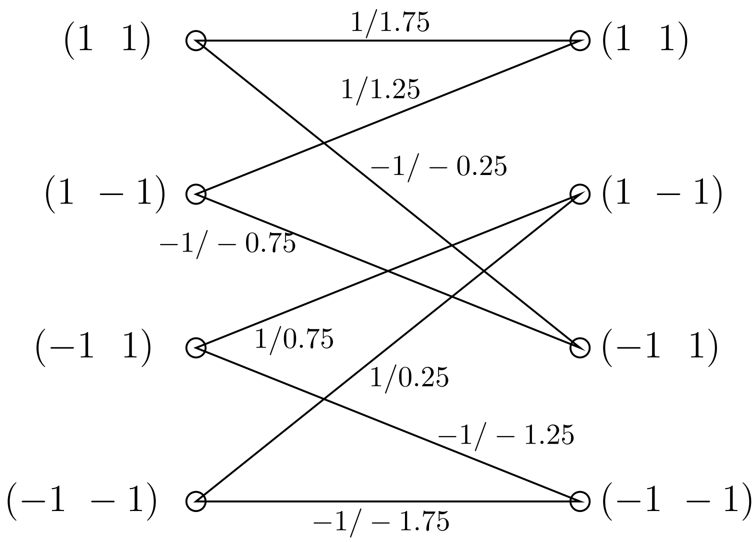

Figure 3:

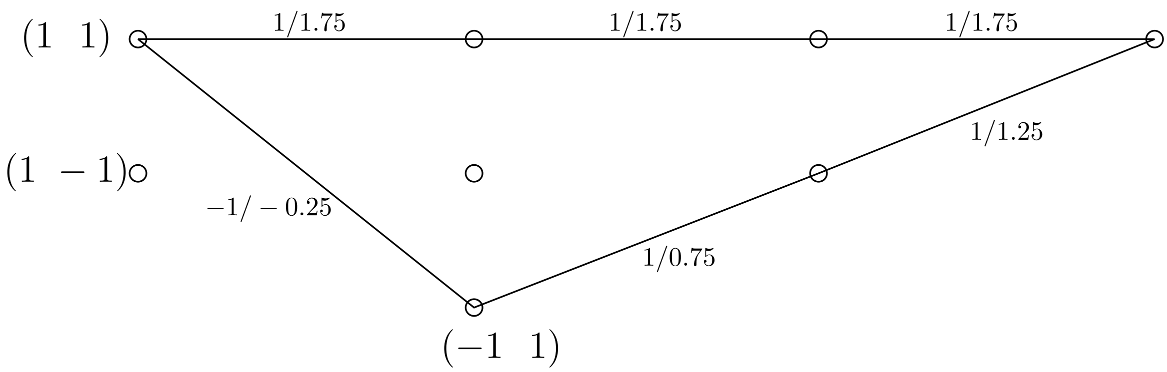

Figure 4: