EE5130: Detection and Estimation Theory
Problem Set 7

1. (Poor IV.F.1) Suppose Θ is a random parameter and that, given $\Theta = \theta$, the real observation Y has density

$$p_\theta(y) = (\theta/2)e^{-\theta|y|}, \quad y \in \mathbf{R}.$$

Suppose further that Θ has prior density

$$w(\theta) = \begin{cases} 1/\theta, & 1 \le \theta \le e \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the MAP estimate of Θ based on Y.

   (b) Find the MMSE estimate of Θ based on Y.

2. (Poor IV.F.2) Suppose we have a real observation Y given by

$$Y = N + \theta S$$

where $N \sim \mathcal{N}(0,1)$, $P(S=1) = P(S=-1) = 1/2$, and Θ has pdf

$$w(\theta) = \begin{cases} Ke^{\theta^2/2}, & 0 \le \theta \le 1 \\ 0, & \text{otherwise} \end{cases}$$

where $K = \left[\int_0^1 e^{\theta^2/2} d\theta\right]^{-1}$. Assume that N, Θ, and S are independent.

   (a) Find the MMSE estimate of Θ given $Y = y$.

   (b) Find the MAP estimate of Θ given $Y = y$.

3. The IID observations Y_n for $n = 0, 1, \cdots, N-1$ have the Rayleigh PDF

$$p_{\sigma^2}(y_n) = \begin{cases} \dfrac{y_n}{\sigma^2} \exp\left(-\dfrac{1}{2}\dfrac{y_n^2}{\sigma^2}\right) & y_n > 0 \\ 0 & y_n < 0 \end{cases}.$$

   Find a sufficient statistic for σ^2.

4. The IID observations Y_n for $n = 0, 1, \cdots, N-1$ are distributed according to $\mathcal{N}(\theta, \theta)$ where $\theta > 0$. Find a sufficient statistic for θ.

5. If $Y_n = A + W_n$ for $n = 0, 1, \cdots, N-1$ are observed, where W_n is WGN with variance σ^2, find the MVUE for σ^2 assuming that A is known. You may assume that the sufficient statistic is complete.

6. Assume thet Y_n is the result of a Bernoulli trial (a coin toss) with

$$\Pr\{Y_n = 1\} = \theta$$

$$\Pr\{Y_n = 0\} = 1 - \theta$$

and that N IID observations have been made. Find a sufficient statistic for θ. Then, assuming completeness, find the MVUE of θ.

7. If N IID observations are made according to the PDF

$$p_\theta(y) = \begin{cases} \exp\left[-(y-\theta)\right] & y > \theta \\ 0 & y < \theta \end{cases},$$

find the MVUE for θ. Note that θ represents the minimum value that Y_n may attain. Assume that the sufficient statistic is complete.