EE5130: Detection and Estimation Theory
Problem Set 6

1. (Poor III.F.20) Derive

$$
\begin{aligned}
P_e &\leq \pi_0 e^{-s\tau} \int_{\Gamma_1} L^s p_0 d\mu + \pi_1 e^{(1-s)\tau} \int_{\Gamma_0} L^s p_0 d\mu \\
\text{and} & \\
P_e &\leq \max\{\pi_0, \pi_1 e^{\tau}\} \exp\{\mu_{T,0}(s) - s\tau\},\ 0 \leq s \leq 1.
\end{aligned}
$$

2. (Poor III.F.22) Consider the hypothesis pair

$$
\begin{aligned}
H_0 &: Y_k = N_k - S_k \\
H_1 &: Y_k = N_k + S_k
\end{aligned}
$$

   where $N_1, ..., N_n$ are i.i.d. Laplacian random variables and where $s_1, ..., s_n$ is a known signal satisfying $s_k \geq \Delta > 0$ for all k and some constant Δ. Show that the minimum error probability in deciding H_0 versus H_1 approaches zero as $n \to \infty$(Δ is independent of n).

3. (Poor III.F.23) Consider the problem of detecting a $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma_S})$ signal in $\mathcal{N}(\mathbf{0}, \sigma^{\mathbf{2}}\mathbf{I})$ noise with $n = 2$ and

$$
\Sigma_S = \sigma^2{}_S \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}
$$

   For the equally likely priors compute and compare the exact error probability and the Chernoff bound on the exact error probability for $\rho = 0.0$, $\rho = -0.5$ and $\rho = +0.5$, and for $\sigma^2{}_S/\sigma^2 = 0.1$, $\sigma^2{}_S/\sigma^2 = 1.0$, and $\sigma^2{}_S/\sigma^2 = 10.0$.

4. (Poor III.F.25) Consider a sequence of i.i.d. Bernoulli observations, $Y_1, Y_2, ...$, with distribution

$$
P(Y_k = 1) = 1 - P(Y_k = 0) = 1/3
$$

   under hypothesis H_0, and

$$
P(Y_k = 1) = 1 - P(Y_k = 0) = 2/3
$$

   under Hypothesis H_1.

   (a) Use Wald's approximations to suggest values of A and B so that the SPRT (A,B) has maximum error probability $p^* = \max(P_F, P_M)$ approximately equal to 0.01. Describe the resulting test in detail. Also, using Wald's approximations, give an approximation to the expected sample sizes $E\{N|H_0\}$ and $E\{N|H_1\}$.

   (b) Find an integer n as small as you can so that the maximum error probability for the optimum test with fixed sample size n is no more than 0.01. Compare n to the expected sample sizes found in part (a) (Note: You may use a Chernoff bound to find n, rather than finding the smallest possible n.)