# EE5130: Detection and Estimation Theory
## Problem Set 3

1. (Poor II.F.3) Suppose $Y$ is a random variable that, under hypothesis $H_j$, has pdf given by

$$p_j(y) = \frac{j+1}{2} e^{-(j+1)|y|}, \; y \in \mathbb{R}, \; j = 0, 1$$

   Find the Neyman-Pearson rule and the corresponding detection probability for false alarm probability $\alpha \in (0, 1)$

2. (Poor II.F.7)

   (a) Consider the hypothesis pair

$$\begin{aligned} H_0 : Y &= N \\ H_1 : Y &= N + S \end{aligned}$$

   where $N$ and $S$ are independent random variables each having pdf

$$p(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

   Find the likelihood ratio between $H_0$ and $H_1$

   (b) Find the threshold and detection probability for $\alpha$-level Neyman-Pearson testing in (a).

   (c) Consider the hypothesis testing pair

$$\begin{aligned} H_0 : Y_k &= N_k, \; k = 1, 2..n \\ H_1 : Y_k &= N_k + S, \; k = 1, 2..n \end{aligned}$$

   where $N \geq 1$ and $N_1, N_2, ...N_n$, and $S$ are independent random variables each having pdf given in (a).
   Find the likelihood ratio.

   (d) Find the threshold for $\alpha$-level Neyman-Pearson testing in $(c)$.

3. (Poor II.F.9) Suppose we have a real observation $Y$ and binary hypothesis described by the following pair of pdfs:

$$p_0(y) = \begin{cases} 1 - |y|, & |y| \leq 1 \\ 0, & |y| > 1 \end{cases}$$

$$p_1(y) = \begin{cases} \frac{(2-|y|)}{4}, & |y| \leq 2 \\ 0, & |y| > 2 \end{cases}$$

   Find the Neyman-Pearson test of $H_0$ versus $H_1$ with false alarm probability $\alpha$. Find the corresponding power of the test.