# EE 511 Problem Set 7

Due on 16 Nov 2007

1. Let $\hat{X}_t$ be the Hilbert transform of the W.S.S. random process $X_t$. Show that (i) $S_{\hat{X}}(f) = S_X(f)$, (ii) $S_{X\hat{X}}(f) = -S_{\hat{X}X}(f)$, (iii) $E[X_t\hat{X}_t] = 0$, and (iv) for $Z_t = X_t + j\hat{X}_t$, determine $S_Z(f)$ in terms of $S_X(f)$.

2. The power spectrum of a W.S.S. band-pass random process $X_t$ is as shown in Figure 1. Sketch $S_{X^I}(f)$, $S_{X^Q}(f)$ and $S_{X^IX^Q}(f)$ assuming 5 MHz to be the carrier frequency.

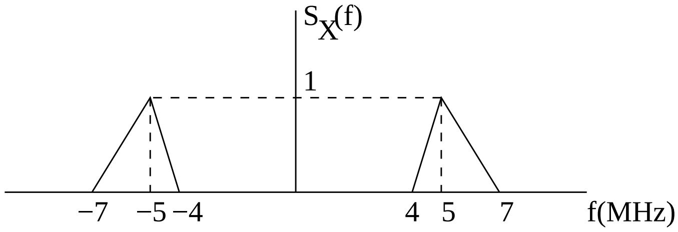


Figure 1:

3. Repeat problem 2 assuming 4 MHz to be the carrier frequency.

4. A narrow-band noise process $N_t$ has zero-mean and auto-correlation function $R_N(\tau)$. Its power spectral density $S_N(f)$ is centered about $\pm f_c$. The in-phase and quadrature components $N_t^I$ and $N_t^Q$ are defined by $N_t^I = N_t \cos 2\pi f_c t + \hat{N}_t \sin 2\pi f_c t$ and $N_t^Q = \hat{N}_t \cos 2\pi f_c t - N_t \sin 2\pi f_c t$. Show that $R_{N^I}(\tau) = R_{N^Q}(\tau) = R_N(\tau)\cos 2\pi f_c\tau + \hat{R}_N(\tau)\sin 2\pi f_c\tau$ and $R_{N^IN^Q}(\tau) = -R_{N^QN^I}(\tau) = R_N(\tau)\sin 2\pi f_c\tau - \hat{R}_N(\tau)\cos 2\pi f_c\tau$, where $\hat{R}_N(\tau)$ is the Hilbert transform of $R_N(\tau)$.

5. A pair of noise processes $N_t$ and $W_t$ are related by $N_t = W_t\cos(2\pi f_c t + \Theta) - W_t \sin(2\pi f_c t + \Theta)$, where $f_c$ is a constant and $\Theta$ is a uniform random variable in the interval $[0, 2\pi]$. The noise process $W_t$ is stationary and its power spectral density is as shown in Figure 2. Find and plot the power spectral density of $N_t$, $S_N(f)$.

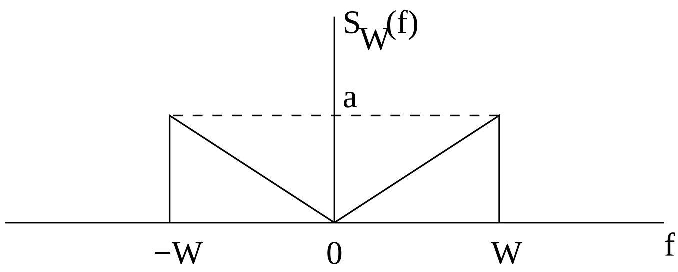


Figure 2:

6. A W. S. S. band-pass random process $X_t$ is generated from two W. S. S. low-pass random processes $X_t^I$ and $X_t^Q$ as follows: $X_t = X_t^I \cos 2\pi f_c t - X_t^Q \sin 2\pi f_c t$. $S_{X^I}(f)$ and $S_{X^IX^Q}(f)$ are as shown in Figure 3. (a) Sketch $S_{X^Q}(f)$ and $S_{X^QX^I}(f)$. (b) Determine and sketch $S_X(f)$

assuming $f_c = 10MHz$. (c) Determine and sketch $S_{\tilde{X}}(f)$, the power spectral density of the complex baseband equivalent process given by $X_t^I + jX_t^Q$.

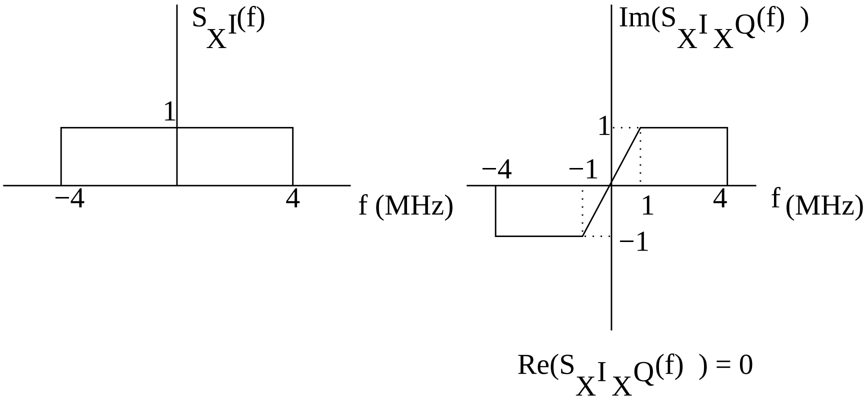


Figure 3:

7. The power spectral density of a wide-sense stationary band-pass random process $X_t$ is as shown in Figure 4.

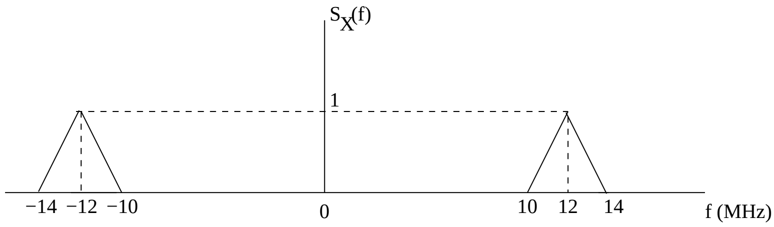


Figure 4:

(a) Assuming that the carrier frequency is 11 MHz, sketch $S_{X^I}(f)$, the power spectral density of the in-phase component of the band-pass process, and mark the important details.
(b) Assuming that the carrier frequency is 11 MHz, sketch $S_{X^I X^Q}(f)$, the cross power spectral density of the in-phase and quadrature components of the band-pass process, and mark the important details.
(c) How are $S_{X^Q}(f)$ and $S_{X^Q X^I}(f)$ related to $S_{X^I}(f)$ and $S_{X^I X^Q}(f)$?