

EE 511 Problem Set 6

Due on 5 November 2007

1. White noise of power spectral density $N_0/2$ is filtered using an ideal low pass filter of bandwidth B . What is the variance of the output noise process?
2. Suppose X_t is a Wiener process defined for $t \geq 0$, i.e., a Gaussian process with $m_X(t) = mt$ for $t \geq 0$, where m is a constant, and $R_X(t, s) = \sigma^2 \min(t, s) + m^2ts$ for $t, s \geq 0$. Define a process $Y_t = X_{t+D} - X_t$ for $t \geq 0$, where D is a fixed positive number. a) Find $m_Y(t)$ and $R_Y(t, s)$. b) Show that Y_t is stationary and find $S_Y(f)$.
3. Let X_t be a zero-mean stationary Gaussian process with auto-correlation function $R_X(\tau)$. This process is applied to a square-law device defined by $Y_t = X_t^2$. a) Show that $E[Y_t] = R_X(0)$. b) Show that the auto-covariance function of Y_t , $C_Y(\tau) = 2R_X^2(\tau)$.
4. A stationary Gaussian process X_t with zero-mean and power spectral density $S_X(f)$ is applied to a linear filter with impulse response as shown in Figure 1. A sample Y is taken of the random process at the filter output at time T . a) Determine the mean and variance of Y . b) What is the probability density function of Y ?

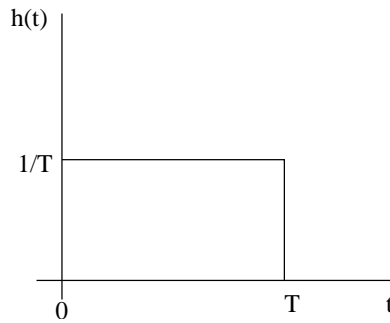


Figure 1:

5. Consider the cascaded filters shown in Figure 2 with a W. S. S. random process X_t as input. (a) Calculate the power spectral densities $S_Y(f)$ and $S_Z(f)$. (b) Calculate $S_{XY}(f)$, $S_{XZ}(f)$, and $S_{ZY}(f)$. (c) Evaluate your answers when the input is zero-mean, white Gaussian noise and $h(t) = e(t) = e^{-t}$ for $t \geq 0$ and $h(t) = e(t) = 0$ for $t < 0$.

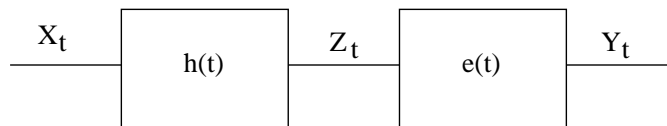


Figure 2:

6. A stationary Gaussian random process X_t with mean m_X and auto-correlation function $R_X(\tau)$ is filtered by $h_1(t)$ to obtain Y_t and $h_2(t)$ to obtain Z_t , respectively. Determine the cross-correlation function $R_{YZ}(t_1, t_2)$ and the cross-spectral density $S_{YZ}(f)$. Assuming $m_X = 0$, under what conditions on the filters are Y_t and Z_t independent? See Figure 3.

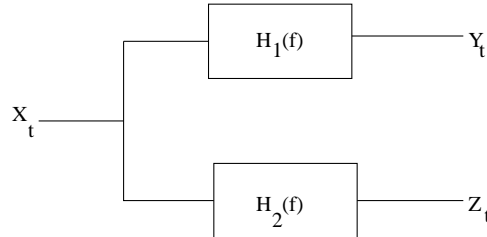


Figure 3:

7. A stationary zero-mean Gaussian random process X_t is filtered by the filter $H_1(f)$ to obtain Y_t , and by the filter $H_2(f)$ to obtain Z_t (See Figure 4). The power spectral density of X_t and the transfer functions $H_1(f)$ and $H_2(f)$ are as shown in Figure 4.

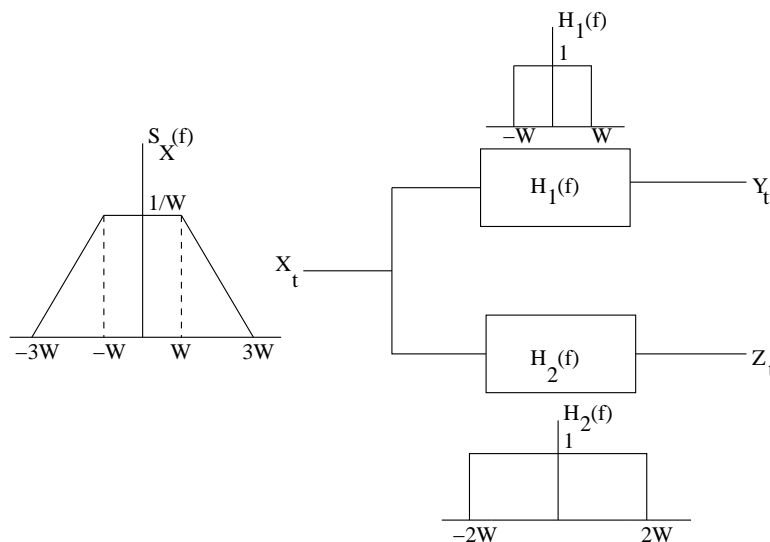


Figure 4:

- (a) Determine $E[Y_t^2]$ and $E[Z_t^2]$.
 (b) Determine $R_{YZ}(\tau)$.
 (c) Write the joint probability density function of Y_{t_1} and Z_{t_2} , when $t_1 = t_2 + (1/2W)$.
8. Consider the Wiener process X_t defined for $t \geq 0$, i. e., a Gaussian process with $E[X_t] = mt$ and $R_X(t, s) = m^2ts + \sigma^2 \min(t, s)$ for $t, s \geq 0$. Define the process $Y_t = c^{-1}X_{c^2t}$ where $c > 0$ is a given constant. Show that Y_t is also a Wiener process and find its mean and auto-correlation function.