EE 511 Problem Set 1

Due on 27 Aug 2007

1. From the axioms of probability, derive for any event A, (i) $P(\overline{A}) = 1 - P(A)$, (ii) $P(A) \leq 1$, (iii) $P(\phi) = 0$. A collection $\{A_i\}_{i=1}^n$ is a partition of the sample space S. (iv) Prove for any event B, $P(B) = \sum_{i=1}^n P(BA_i)$.

2. If $A \subset B$, show that $P(A) \leq P(B)$.

3. Prove the following identity: $P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$.

4. For arbitrary events $\{A_i\}_{i=1}^n$, prove $P\left(\sum_{i=1}^n A_i\right) \leq \sum_{i=1}^n P(A_i)$.

5. If $P(A) \geq 1 - \delta$ and $P(B) \geq 1 - \delta$, prove that $P(AB) \geq 1 - 2\delta$, i.e., if A and B are events with probability nearly one, so is AB.

6. If $A \subset B$, $P(A) = 1/4$ and $P(B) = 1/3$, find $P(B|A)$ and $P(A|B)$.

7. A, B and C are three events. Show that $P(AB|C) = P(A|BC)P(B|C)$ and $P(ABC) = P(A|BC)P(B|C)P(C)$.

8. A, B and C are three events such that (i) $P(A) > P(B) > P(C) > 0$, (ii) A and B partition the sample space S and (iii) A and C are independent. Can B and C be disjoint ?

9. If two events A and B are independent, show that (i) $\overline{A}$ and B are independent and (ii) $\overline{A}$ and $\overline{B}$ are independent.

10. If three events A, B, and C are independent, show that the events A and $B + C$ are independent.