EC305 Problem Set 1

1. Let $x(t) = m(t)\cos 2\pi f_c t$, where $m(t)$ is a real lowpass signal with bandwidth W and $f_c > W$. Find $\hat{x}(t)$, the Hilbert transform of $x(t)$.

2. If $x(t)$ is a real signal with no impulses in its spectrum at $f = 0$, and $\hat{x}(t)$ is its Hilbert transform, show the following:

   (a) $x(t)$ and $\hat{x}(t)$ have the same energy.

   (b) The Hilbert transform of $\hat{x}(t)$ is $-x(t)$.

   (c) $x(t)$ and $\hat{x}(t)$ are orthogonal.

   (d) The Hilbert transform of $x(\alpha t)$, where α is a non-zero constant, is $\text{sgn}(\alpha)\hat{x}(\alpha t)$.

3. Consider a bandpass signal $x(t) = m_1(t)\cos 2\pi f_c t - m_2(t)\sin 2\pi f_c t$.

   (a) Determine the in-phase and quadrature components of this signal when the local oscillators used have a phase offset of θ, i.e., they are $\cos(2\pi f_c t + \theta)$ and $\sin(2\pi f_c t + \theta)$.

   (b) Specialise the result to the case when $m_2(t) = 0$. Compare this with part (a).

4. Consider the non-ideal Hilbert transformer in the figure below. If $m(t)$ is the input, and $\hat{m}'(t)$ the output, sketch the spectrum of $x(t) = m(t)\cos 2\pi f_c t + \hat{m}'(t)\sin 2\pi f_c t$, where $m(t)$ is a lowpass signal with bandwidth W. What is the bandwidth of $x(t)$?

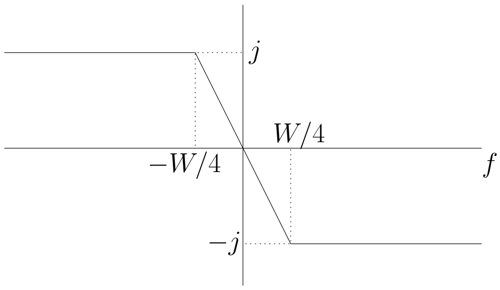

Figure 1: Non-ideal Hilbert transformer

5. Consider a non-linear device whose transfer characteristic is defined by $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$, where a_1 and a_2 are constants, $v_1(t)$ is the input, and $v_2(t)$ is the output. Suppose $v_1(t) = A_c[1 + g_m m(t)]\cos 2\pi f_c t$ is an AM signal.

   (a) Evaluate the output $v_2(t)$.

   (b) Find the conditions for which the message signal $m(t)$ may be recovered from $v_2(t)$.

6. Suppose an AM signal $s(t) = A_c[1 + g_m m(t)]\cos 2\pi f_c t$, where $m(t)$ is a lowpass signal with bandwidth W. Let $v_1(t) = s^2(t)$, $v_2(t)$ be a low pass filtered version of $v_1(t)$ (LPF with bandwidth $2W$), and $v_3(t) = \sqrt{v_2(t)}$. Assuming $|g_m m(t)| < 1$ for all t, determine whether $m(t)$ can be obtained from $v_3(t)$.

7. Derive the time-domain equation for the SSB signal corresponding to the lower side band of an AM signal $m(t)\cos 2\pi f_c t$.