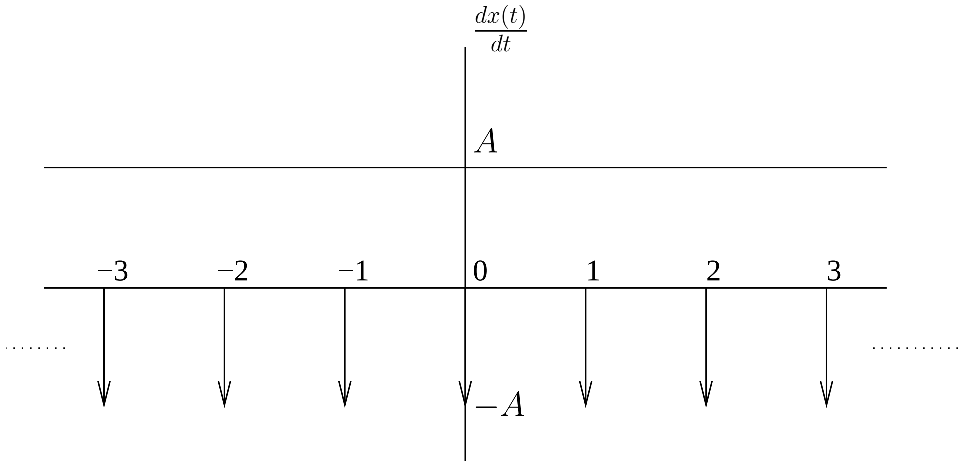
EC204: Networks & Systems
Solutions to Problem Set 3

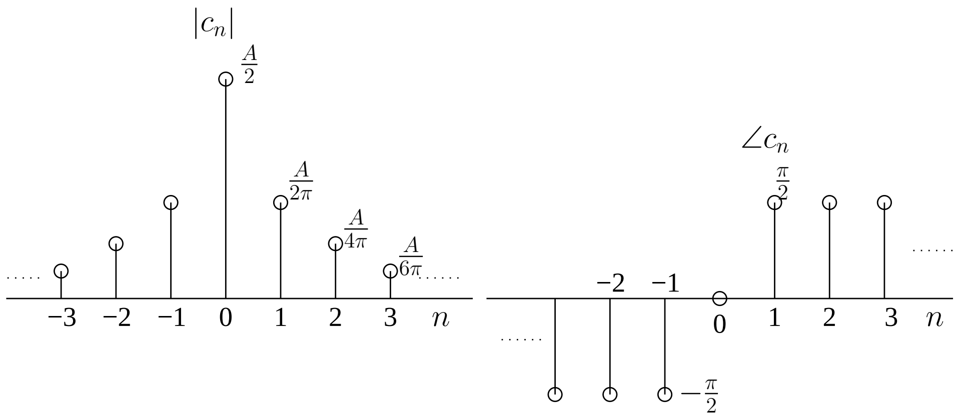
1. (a) The period of $x(t)$ is 1. Therefore, $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt}$. The d.c. value $c_0 = A/2$. Now, let the first derivative of $x(t)$ (shown in figure below) be expressed as $\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} d_n e^{j2\pi nt}$.



It can be easily determined that $d_n = -A$ for $n \neq 0$ and $d_0 = 0$. Using this result, we can determine c_n as

$$c_n = \frac{d_n}{j2\pi n} = \frac{jA}{2\pi n}.$$

The magnitude and phase spectrum of $x(t)$ are shown below.



(b) $y(t)$ can be expressed in terms of $x(t)$ as $y(t) = x(-t+0.5) + A$. Therefore, the Fourier coefficients of $y(t)$ are

$$c_0 = \frac{3A}{2} \quad \text{and} \quad c_n = \frac{-jA}{2\pi n} e^{-j2\pi n(0.5)} = \frac{-jA}{2\pi n}(-1)^n \quad \text{for } n \neq 0.$$

2. The output $y(t) = x(t) \star h(t)$, where $\star$ represents convolution.

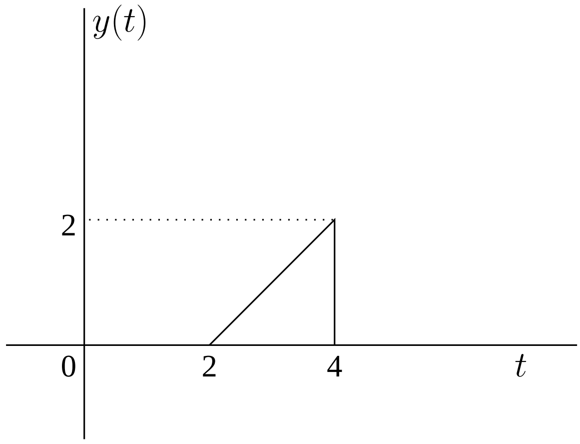
For $t < 2$, $y(t) = 0$. For $2 < t \leq 4$, we have

$$y(t) = \int_0^{t-2} 2e^{-2\tau} d\tau = 1 - e^{-2(t-2)}.$$

Similarly, for $t > 4$, we have

$$y(t) = \int_{t-4}^{t-2} 2e^{-2\tau} d\tau = e^{-2(t-4)} - e^{-2(t-2)}.$$

3. $y(t)$ is shown in the figure below.



4. $(-1)^n = e^{jn\pi}$. Therefore, we have

$$y(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt} e^{jn\pi} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n(t+0.5)} = x(t+0.5) = x(t+0.5+k),$$

where k is any integer.

5. Let $x_1(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$ and $x_2(t) = \sum_{n=-\infty}^{\infty} d_n e^{jn\omega_0 t}$. The Fourier coefficients can be shown to be:

$$c_n = \frac{Ad}{T_0} \text{sinc}\left(n\omega_0 d/2\right),$$

$$d_n = \begin{cases} \frac{A}{\pi} & n = 0 \\ \frac{-A}{\pi(n^2-1)} & n \text{ even}, n \neq 0 \\ \frac{A}{j4n} & n = \pm 1 \\ 0 & n \text{ odd}, |n| \neq 1 \end{cases}.$$

Steps to find d_n:

- Express $x(t) = x_o(t) + x_e(t)$, where $x_o(t)$ and $x_e(t)$ are the odd and even parts of $x(t)$.
- Determine the Fourier series coefficients of $x_o(t) = \frac{A}{2} \sin \omega_0 t$.
- Determine the Fourier series coefficients of $x_e(t)$ (a full-wave rectified sine wave).
- Determine the Fourier series coefficients of $x(t)$ as the sum of the coefficients of $x_o(t)$ and $x_e(t)$.