Optimal Multi-antenna Transmission with Multiple Power Constraints

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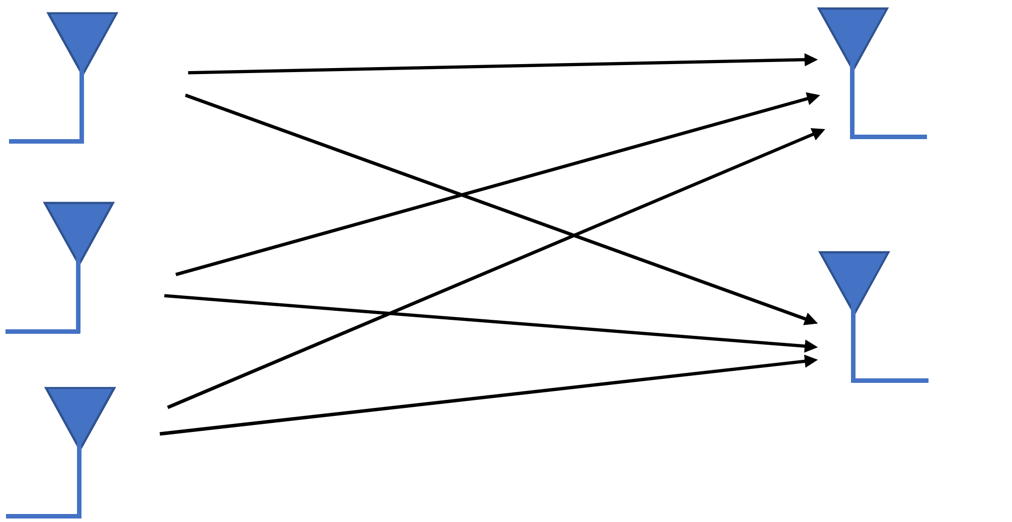
Joint work with Ragini Chaluvadi and Silpa S. Nair

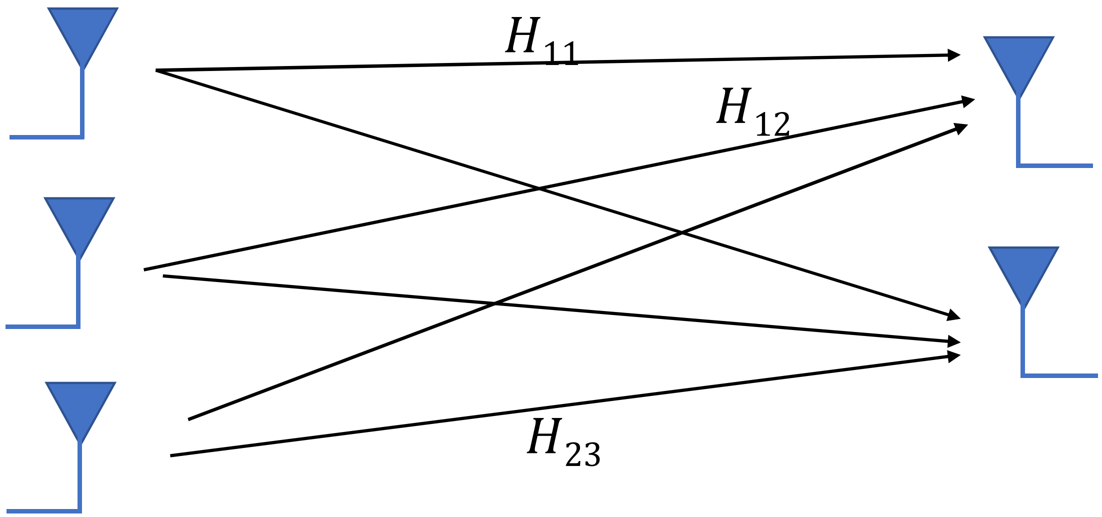
Challenges in Wireless Systems

- Time variations (or) Fading
- Interference



- Multi-antenna systems have additional degrees of freedom
  - Diversity to combat fading
  - Spatial multiplexing to increase rate
  - Interference suppression

Point-to-Point MIMO Systems



$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

$\mathbf{x}$: N_t x 1 transmit vector

$\mathbf{H}$: N_r x N_t channel matrix

$\mathbf{y}$: N_r x 1 received vector

$\mathbf{w}$: N_r x 1 Gaussian noise

Capacity under a sum power constraint

$$\text{Capacity} = \max_{\mathbf{Q}} \log\left|\mathbf{I} + \mathbf{HQH}^{\mathrm{H}}\right|$$

subject to $\mathrm{trace}\{\mathbf{Q}\} \leq P_{tot}$

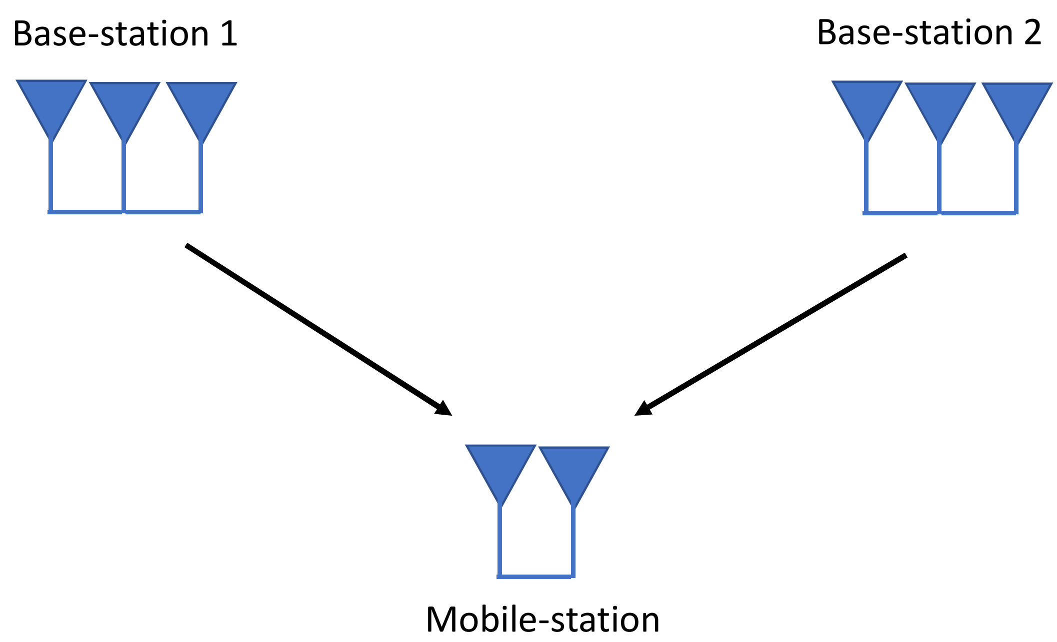
Singular Value Decomposition of $\mathbf{H} = \mathbf{U\Sigma V}^{\mathrm{H}}$

Waterfilling power allocation

E. Telatar, “Capacity of multi-antenna gaussian channels,” European Trans. on Telecomm., vol. 10, no. 6, pp. 585–595, 1999.

Per-antenna/Per-Group power constraints

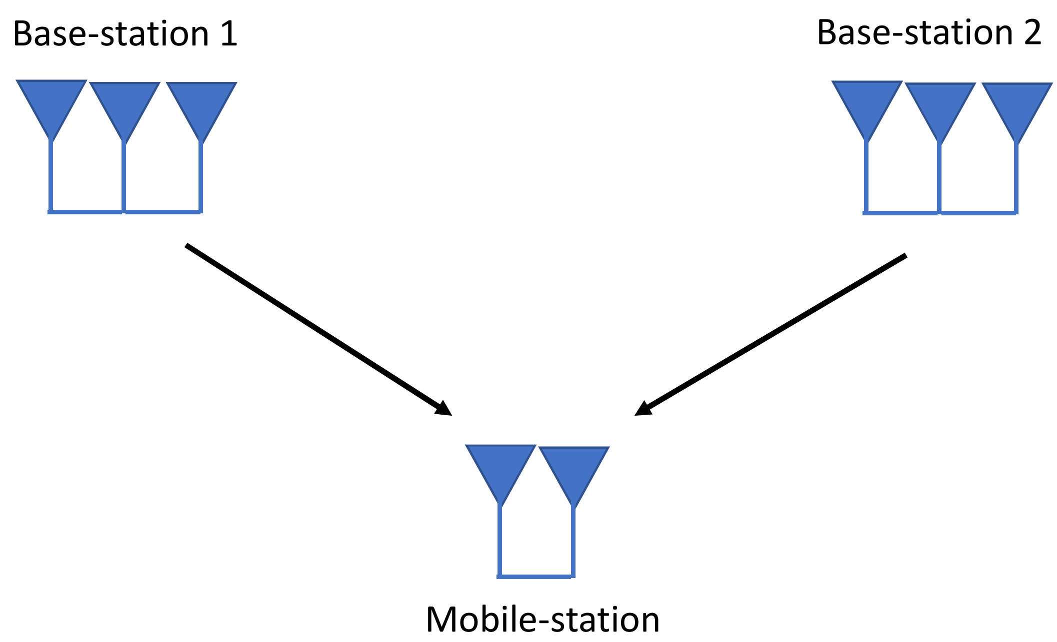
- Distributed antenna systems
  - Coordinated multipoint transmission (CoMP)
  - Cell-free massive MIMO
- Hardware restrictions
  - Power constraint per antenna or group of antennas
- Cutset bound computation



Multiple simultaneous power constraints

- Sum power constraint
  - Regulations
  - Energy efficiency
  - Interference

- Per-antenna/per-group constraints
  - Distributed antennas
  - Hardware restrictions



Capacity under multiple power constraints

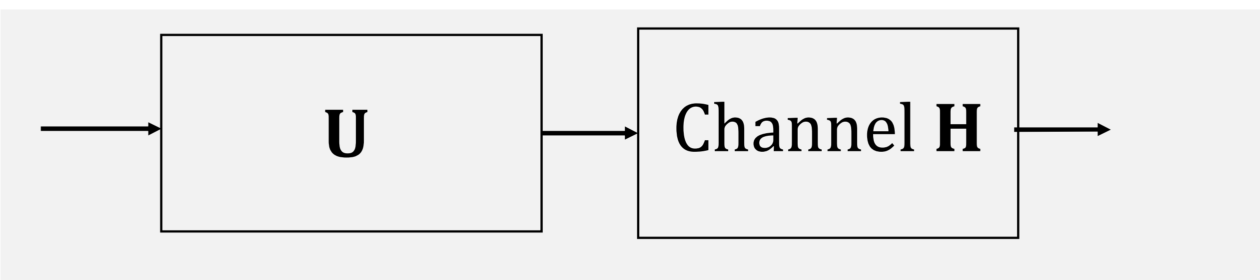
$$\text{Capacity} = \max_{\mathbf{Q}} \log\left|\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\mathrm{H}}\right|$$

subject to

SPC: $\text{trace}\{\mathbf{Q}\} \leq P_{tot}$

PGPC: $\sum_{i \in I(k)} Q_{ii} \leq \tilde{P}_{\mathrm{k}}$

PAPC: $Q_{ii} \leq \hat{P}_i$



$$\mathbf{Q} = \mathbf{U}\mathbf{U}^{\mathrm{H}}$$

No general closed form solution

Convex optimization toolboxes

Known results

- PAPC
    - MISO (closed-form) [Vu 2011]
    - MIMO full rank optimal covariance matrix (closed form) [Tuninetti 2014]
    - MIMO (algorithms) [Vu 2011]

- PGPC
    - MIMO (approximate algorithm) [Xing et al. 2015]

M. Vu, “MISO capacity with per-antenna power constraint,” IEEE Trans. Commn., vol. 59, no. 5, pp. 1268–1274, May 2011.

D. Tuninetti, “On the capacity of the AWGN MIMO channel under per-antenna power constraints,” in Communications (ICC), 2014 IEEE International Conference on, June 2014, pp. 2153–2157.

M. Vu, “The capacity of MIMO channels with per-antenna power constraint,” CoRR, vol. abs/1106.5039, 2011. [Online]. Available: http://arxiv.org/abs/1106.5039

C. Xing, Z. Fei, Y. Zhou, and Z. Pan, “Matrix-field water-filling architecture for mimo transceiver designs with mixed power constraints,” in 2015 IEEE PIMRC, Aug 2015, pp. 392–396.

Known results

- Joint SPC and PAPC
  - MISO (closed form) [Cao et al. 2016, Loyka 2017]
  - MIMO (approximate algorithm) [Cao et al. 2017]

P. L. Cao, T. J. Oechtering, R. F. Schaefer, and M. Skoglund, “Optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints,” IEEE Trans. on Signal Processing, vol. 64, no. 16, pp. 4296–4306, Aug 2016.

S. Loyka, “The capacity of gaussian MIMO channels under total and per-antenna power constraints,” IEEE Transactions on Communications, vol. 65, no. 3, pp. 1035–1043, March 2017.

P. L. Cao and T. J. Oechtering, “Optimal transmit strategy for mimo channels with joint sum and per-antenna power constraints,” in 2017 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), March 2017, pp. 3569–3573.

Our work:
Multiple power constraints SPC-PGPC-PAPC

- Analytical solution
  - MISO
  - Full rank optimal covariance matrix MIMO
  - 2 x N_r

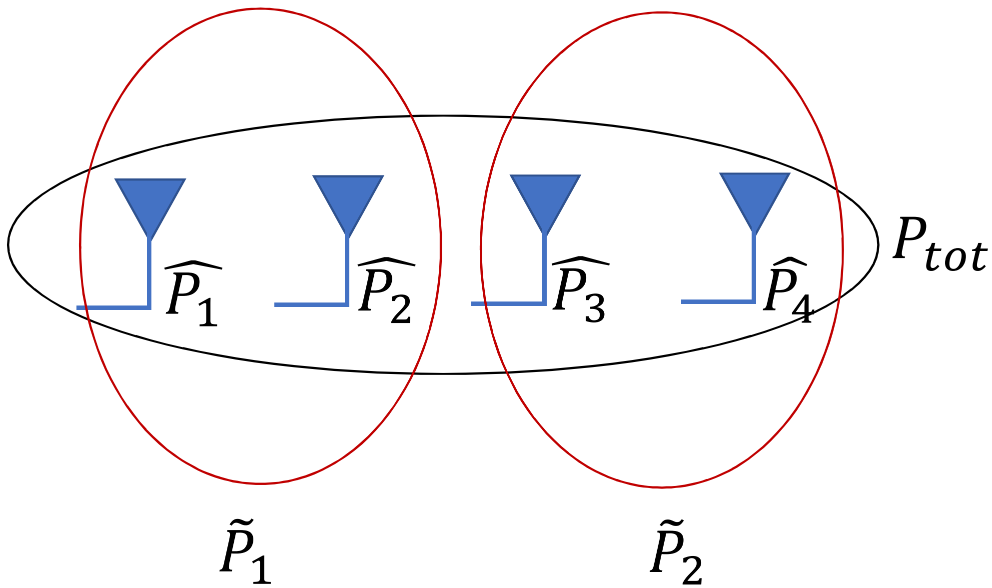
- Projected Factored Gradient Descent (PFGD) algorithm
  - General MIMO under SPC-PGPC-PAPC
  - General MIMO under SPC-PGPC-PAPC + Rank constraint
  - Directly solves for the precoding matrix $\mathbf{U}$

MISO case: Analytical solution

Example

4 transmit antennas, 2 groups of 2 antennas each

$I(1) = \{1, 2\}, I(2) = \{3, 4\}$



$P_{tot} \leq \tilde{P}_1 + \tilde{P}_2$ $\quad$ $\tilde{P}_1 \leq \widehat{P_1} + \widehat{P_2}$ $\quad$ $\tilde{P}_2 \leq \widehat{P_3} + \widehat{P_4}$

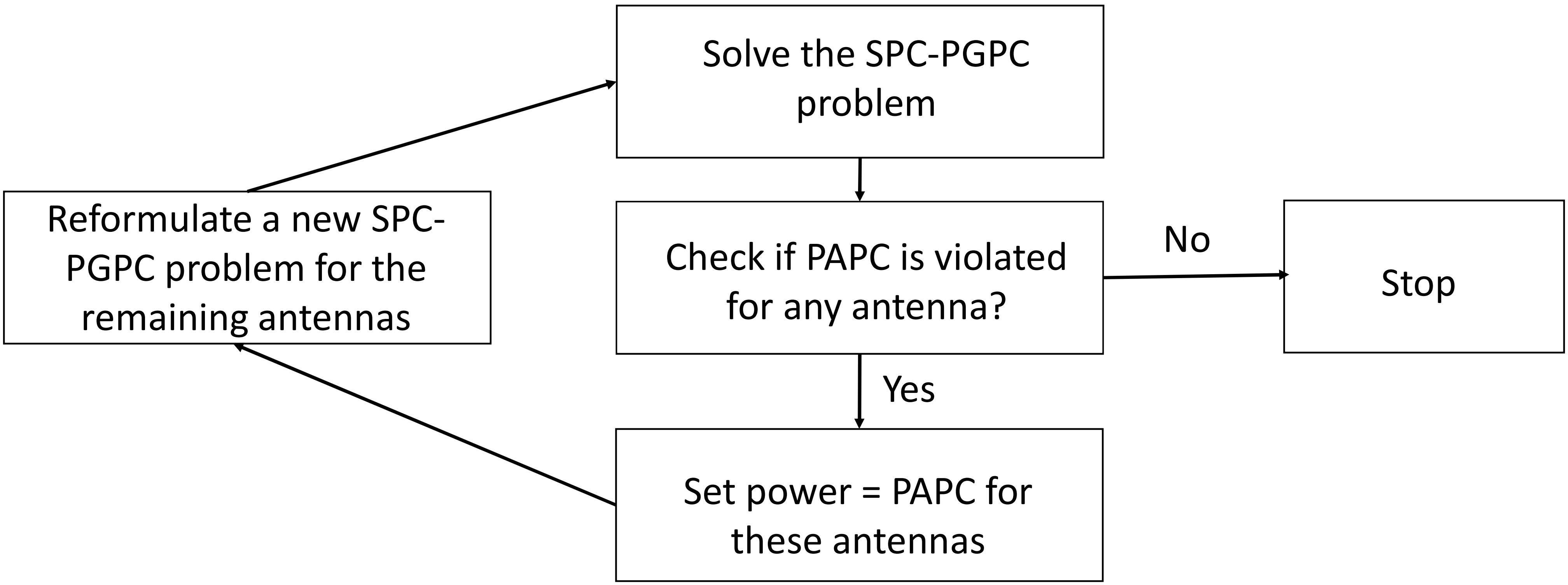
Reduction to a power allocation problem

- Useful observations
  - Rank of optimal covariance matrix ≤ Rank of channel = 1, $\mathbf{Q} = \mathbf{q}\mathbf{q}^{\mathrm{H}}$
  - Align the phase of signal received from each transmit antenna
    - Phase of i^{th} entry of $\mathbf{q} = -$Phase of channel h_i from i^{th} antenna to receiver
  - Optimal solution uses the full sum power P_{tot}
- Problem of finding $\mathbf{Q}$ reduces to a power allocation problem
- Need to solve for power of each antenna $P_1, P_2, \ldots, P_{N_t}$

Solution outline

- Relaxed problem (without PAPC) can be solved in closed form
- If PAPC violated for any antenna, set power = PAPC
- Power decided for at least one antenna in each step
- At most N_t steps

MISO Joint SPC-PGPC-PAPC solution



Atmost N_t steps needed

MISO Joint SPC-PGPC solution

- Order groups such that $\frac{\sum_{j\in I(1)} |h_j|^2}{\tilde{P}_1} \geq \frac{\sum_{j\in I(2)} |h_j|^2}{\tilde{P}_2} \geq \dots \geq \frac{\sum_{j\in I(g)} |h_j|^2}{\tilde{P}_g}$
- Find smallest k such that $\frac{P_{tot} - \sum_{j=1}^{k} \tilde{P}_j}{\sum_{j\in I(i), i\geq k+1} |h_j|^2} \leq \frac{\tilde{P}_{k+1}}{\sum_{j\in I(k+1)} |h_j|^2}$

- Power allocation P_j for $j \in I(i)$ as follows

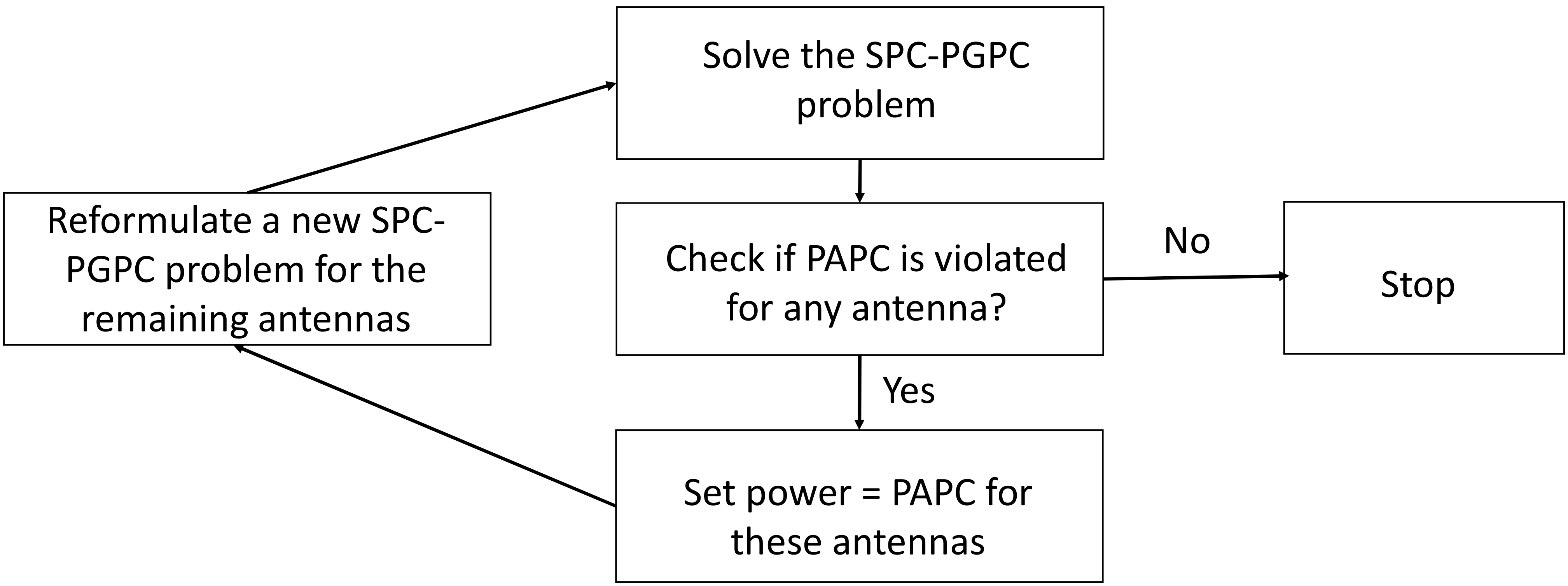
$$P_j = \begin{cases} \frac{\tilde{P}_i}{\sum_{r\in I(i)} |h_r|^2} |h_j|^2 & i = 1, 2, \dots, k \\ \frac{P_{tot} - \sum_{j=1}^{k} \tilde{P}_j}{\sum_{r\in I(i), i\geq k+1} |h_r|^2} |h_j|^2 & i = k+1, \dots, g \end{cases}$$

Remarks on MISO Joint SPC-PGPC solution

- Generalizes closed form solution for Joint SPC-PAPC in [Loyka 2017]
- Solution is Ordering + sequence of SPC solutions as in [Cao 2016]
- Identification of metric for ordering groups is important

Special cases of MIMO: Analytical solution

MIMO: Full rank optimal covariance matrix



MIMO vs MISO

- $\mathbf{A} = (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}$, $a_i = \sum_{j \in I(i)} [(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}]_{\boldsymbol{jj}}$
- Order groups such that $\frac{a_1 + \tilde{P}_1}{n_1} \leq \frac{a_2 + \tilde{P}_2}{n_2} \leq \ \ldots \ \leq \frac{a_g + \tilde{P}_g}{n_g}$
- Optimal $\mathbf{Q} = (\boldsymbol{\Lambda} - \lambda\mathbf{I})^{-1} - \mathbf{A}$

General MIMO: PFGD Algorithm

General MIMO

- Semidefinite program
- Can use generic convex optimization methods
- Capacity $= \max_{\mathbf{Q}} \log\left|\mathbf{I} + \mathbf{HQH}^{\mathrm{H}}\right|$

subject to

SPC: $\text{trace}\{\mathbf{Q}\} \leq P_{tot}$

PGPC: $\sum_{i \in I(k)} Q_{ii} \leq \tilde{P}_{\mathrm{k}}$

PAPC: $Q_{ii} \leq \hat{P}_i$

General MIMO with rank constraints

- $\text{Capacity} = \max_{\mathbf{Q}} \log\left|\mathbf{I} + \mathbf{HQH}^{\mathrm{H}}\right|$

  subject to

  SPC: $\text{trace}\{\mathbf{Q}\} \leq P_{tot}$

  PGPC: $\sum_{i \in I(k)} Q_{ii} \leq \tilde{P}_{\mathrm{k}}$

  PAPC: $Q_{ii} \leq \hat{P}_i$

  $\text{rank}\{\mathbf{Q}\} \leq r$

- Non-convex because of the rank constraint

Reformulation: Convex to Non-convex

- Optimize $\mathbf{U}$, where $\mathbf{Q} = \mathbf{U}\mathbf{U}^{\mathrm{H}}$
  - $\log\left|\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\mathrm{H}}\right| = \log\left|\mathbf{I} + \mathbf{H}\mathbf{U}(\mathbf{H}\mathbf{U})^{\mathrm{H}}\right|$
- Easily enforces positive definiteness and rank constraints
- We anyway need $\mathbf{U}$ as the precoder
- Easier to solve this non-convex problem [Park et al. 2016]

D. Park, A. Kyrillidis, S. Bhojanapalli, C. Caramanis, and S. Sanghavi, "Provable Burer-Monteiro factorization for a class of norm-constrained matrix problems," arXiv preprint arXiv:1606.01316, 2016.

Projected Factored Gradient Descent (PFGD)

- Initialize $\mathbf{U}$, choose size to enforce rank constraint

- Gradient

$$\nabla_{\mathbf{U}} f(\mathbf{U}) = 2\mathbf{H}^{\mathrm{H}}\left(\mathbf{I} + \mathbf{HU}(\mathbf{HU})^{\mathrm{H}}\right)^{-1}\mathbf{HU} = 2\mathbf{H}^{\mathrm{H}}\mathbf{H}\left(\mathbf{I} + \mathbf{U}^{\mathrm{H}}(\mathbf{H}^{\mathrm{H}}\mathbf{H})\,\mathbf{U}\right)^{-1}$$

- Projected gradient descent

$$\mathbf{U}_{k+1} = \mathrm{Projection}(\mathbf{U}_k + \boldsymbol{\eta}\nabla_{\mathbf{U}} f(\mathbf{U}))$$

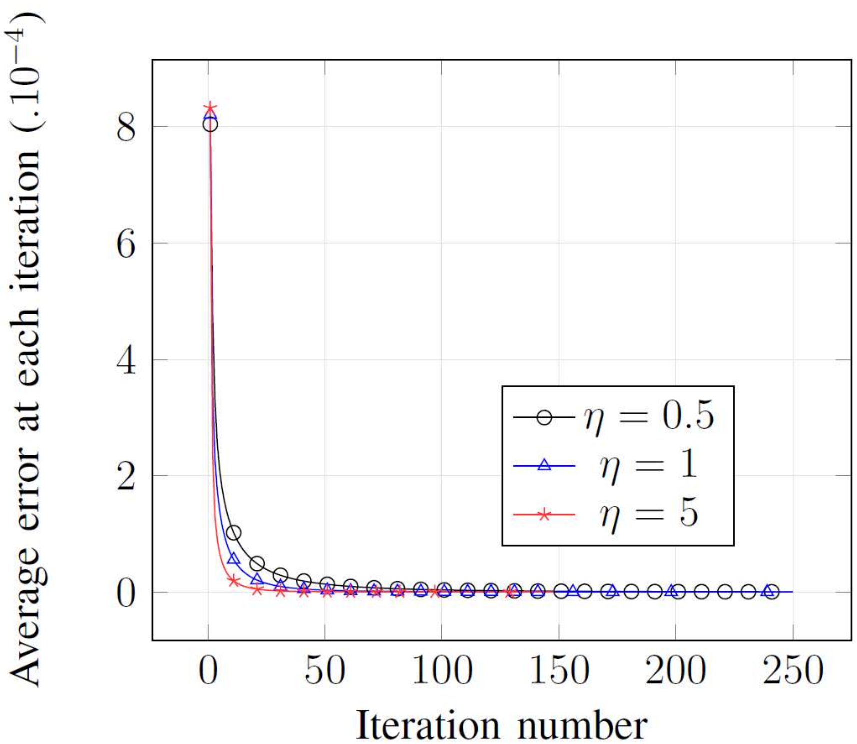
- How to compute the projection?

Projection step

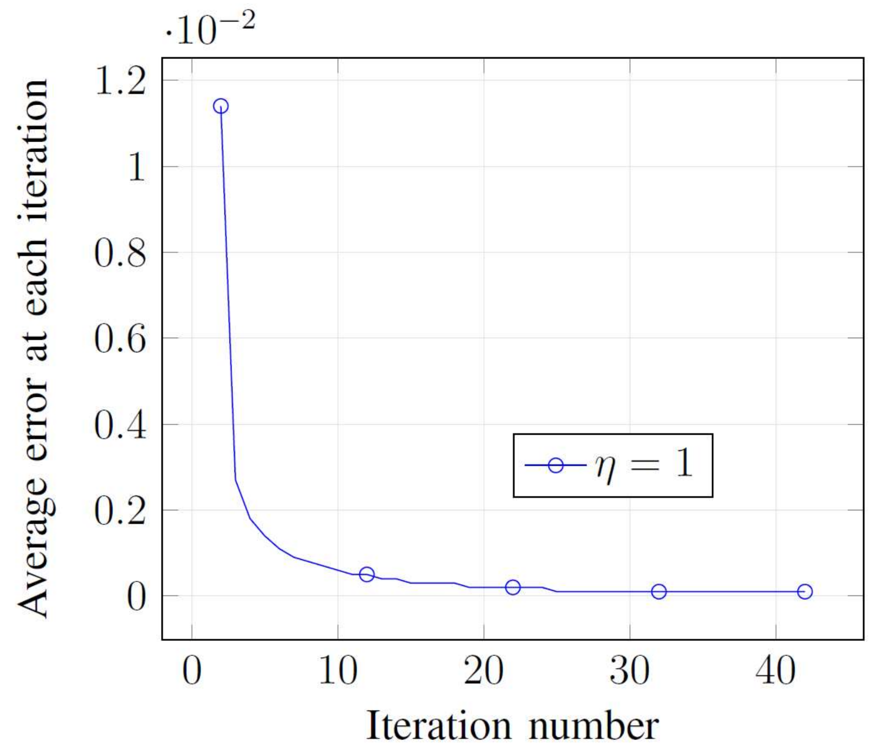
- Closed form solution for projection to SPC-PGPC constraint set
  - Reduces to scaling each row of $\mathbf{U}$ appropriately
- Check if this projection satisfies PAPC
- If not, scale rows that do not satisfy and reduce to a modified SPC-PGPC projection problem

Convergence

- Local convergence result in [Park et al. 2016]



(a) Convergence plot for Joint-SPC-PGPC



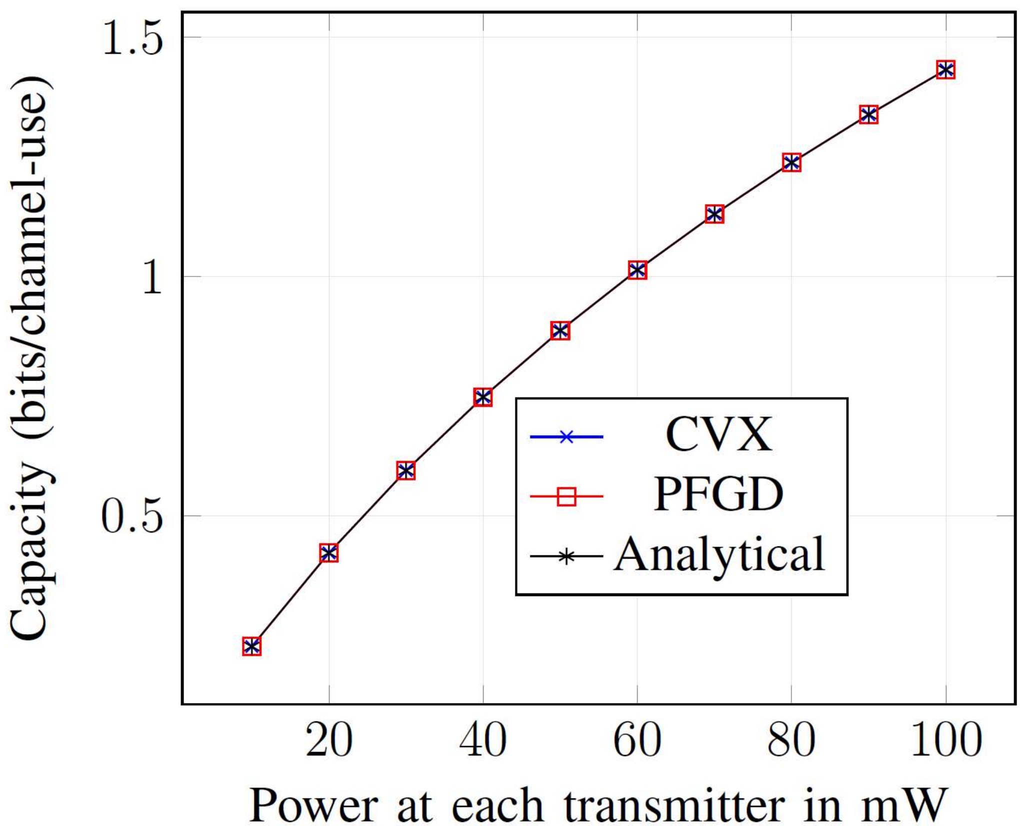
(b) Convergence plot for Joint-SPC-PGPC-PAPC

Numerical Results

Numerical Results

- Compare analytical result with PFGD and CVX (MISO)
- Compare result of PFGD algorithm and CVX toolbox (MIMO)
  - Accuracy
  - Runtime
- Compare with existing algorithms for special cases
  - MIMO PAPC [Vu2011], [Xing2015]
  - MIMO SPC-PAPC [Cao2017]
- Rank-constrained capacity for different rank

MISO SPC-PGPC-PAPC

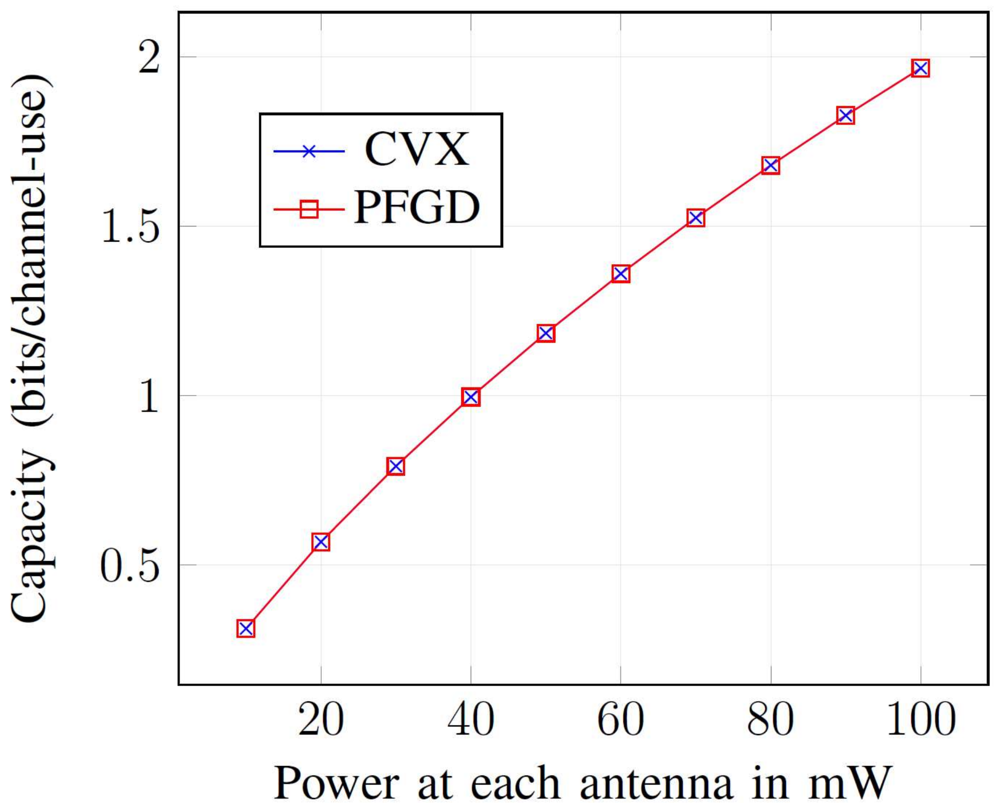


$N_t = 4, N_r = 1$

2 groups, 2 antennas each

$$P_{tot} = \frac{4\hat{P}}{1.21}, \tilde{P}_k = \frac{2\hat{P}}{1.1}, \hat{P}_i = \hat{P}$$

MIMO SPC-PGPC-PAPC



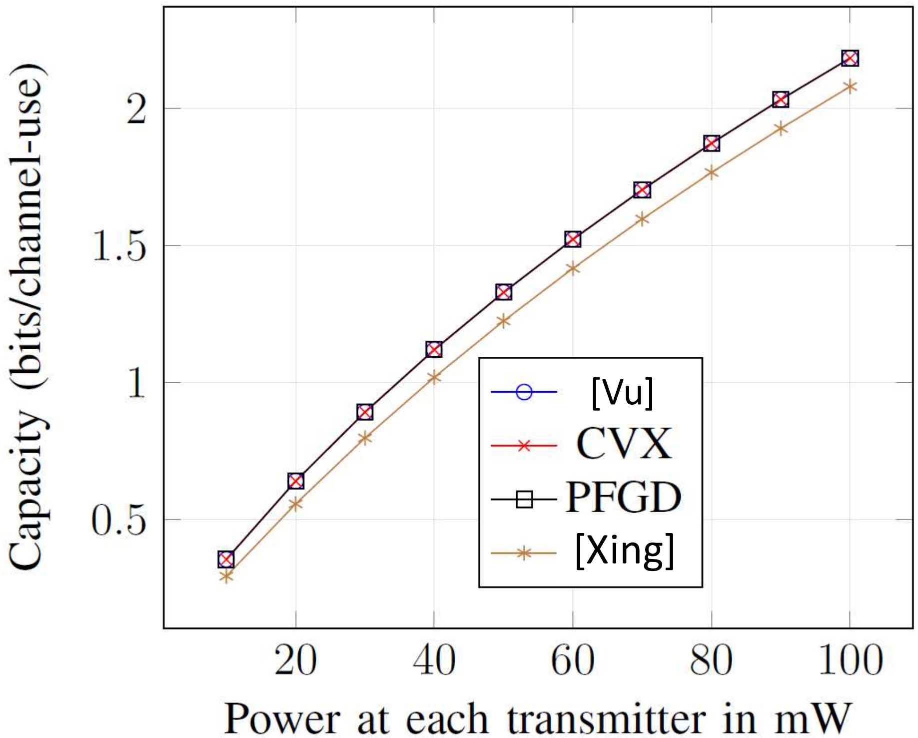
$N_t = 4, N_r = 4$

2 groups, 2 antennas each

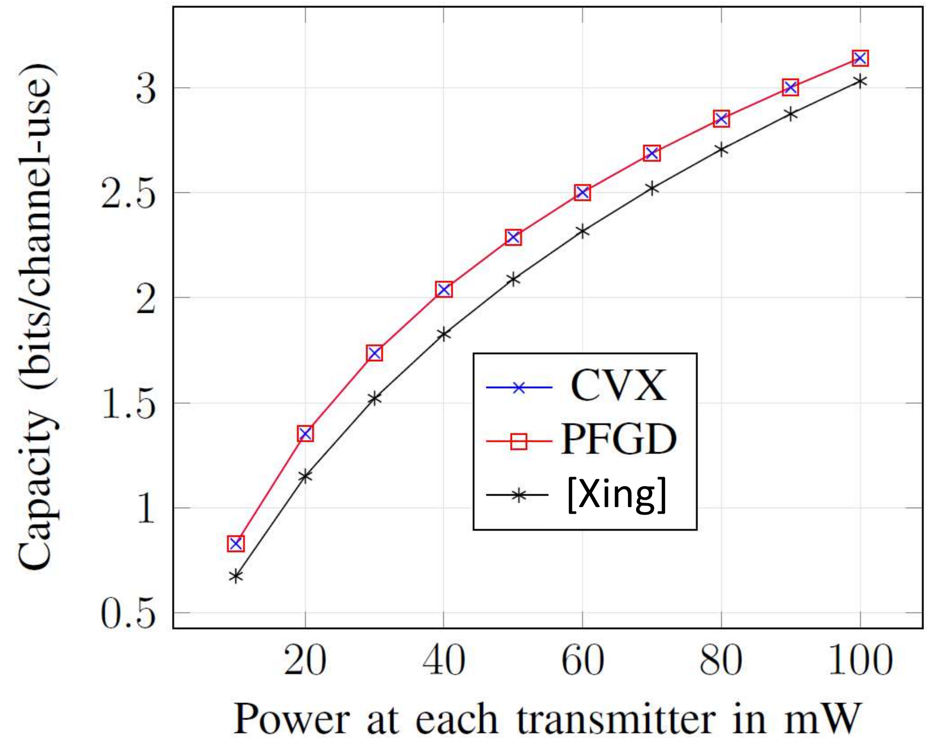
$$P_{tot} = \frac{4\hat{P}}{1.21}, \tilde{P}_k = \frac{2\hat{P}}{1.1}, \hat{P}_i = \hat{P}$$

MIMO PAPC

$N_t = 4, N_r = 1$



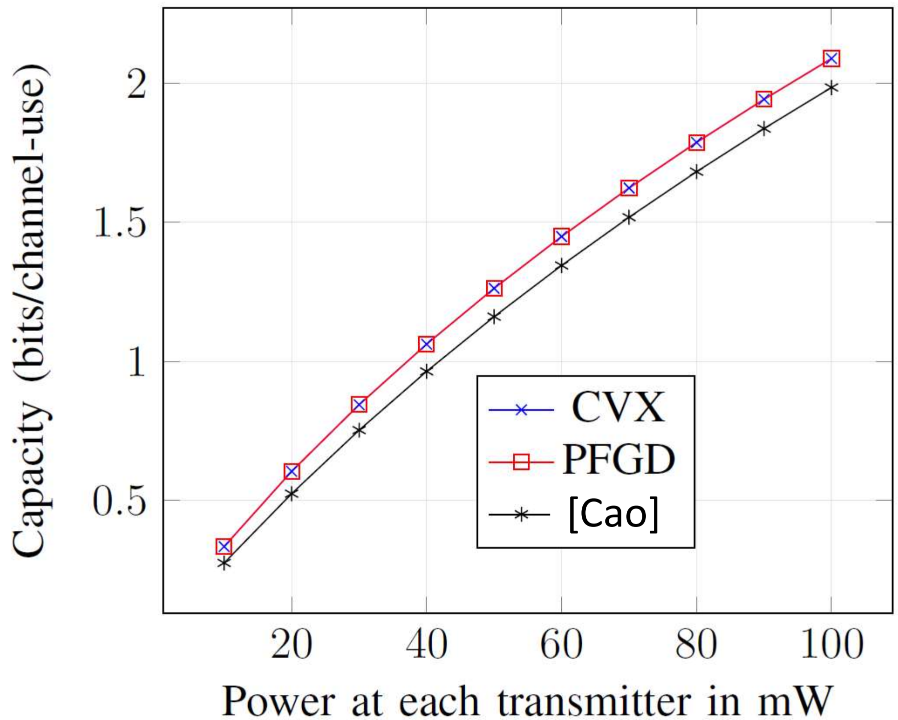
(a) Full rank **H**



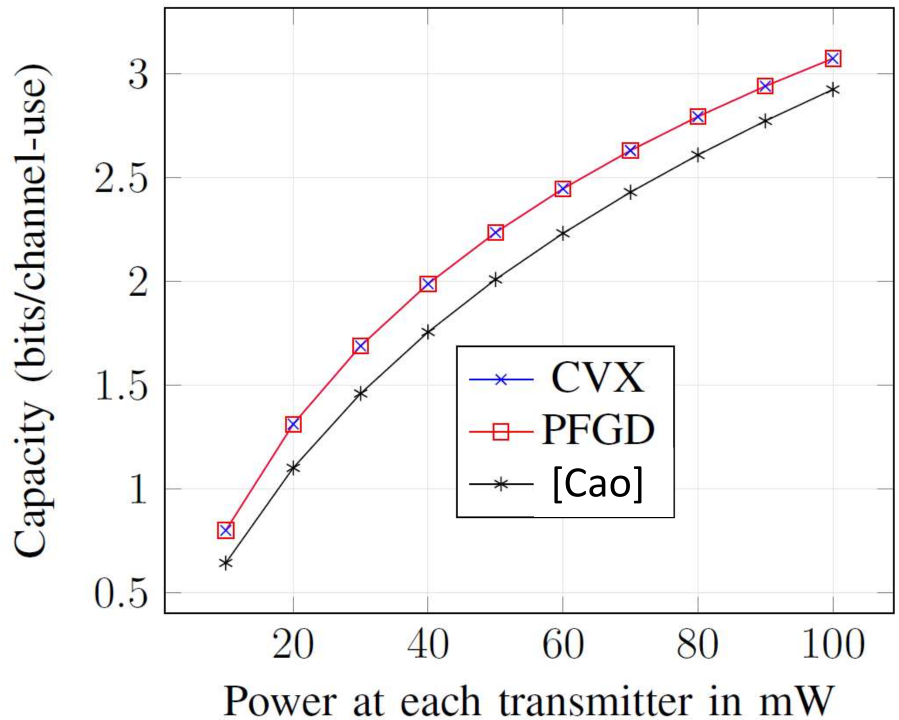
(b) Low rank **H**

MIMO SPC-PAPC

$N_t = 4, N_r = 4 \quad P_{tot} = \frac{4\hat{P}}{1.21}, \hat{P}_i = \hat{P}$

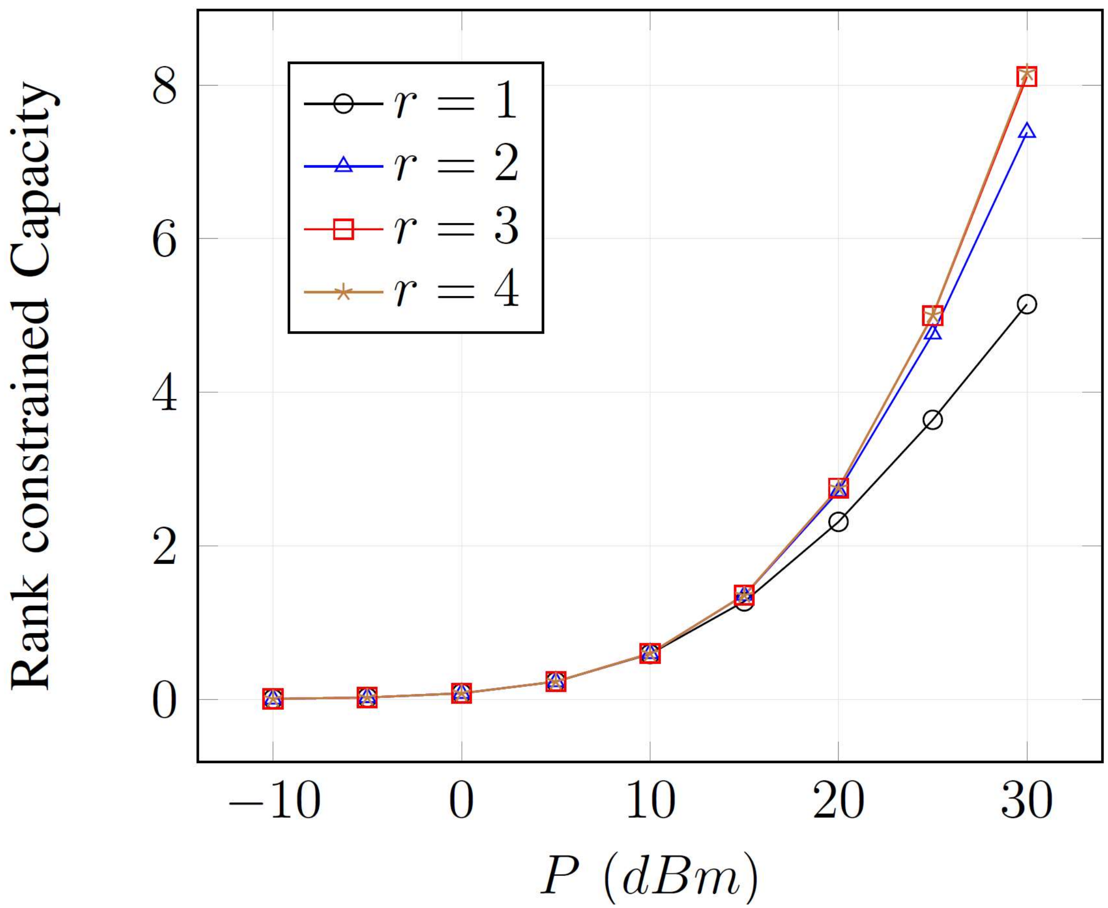


(a) Full rank H



(b) Low rank H

Rank-constrained MIMO SPC-PGPC-PAPC



$N_t = 8, N_r = 8$

4 groups, 2 antennas each

$P_{tot} = \frac{4\tilde{P}_k}{1.2}, \tilde{P}_k = \frac{2\hat{P}}{1.2}, \hat{P}_i = \hat{P}$

Channel rank = 4

Runtime: MIMO SPC-PGPC

$N_t = n_t, N_r = 2$

2 groups with equal number of antennas each

$$P_{tot} = \frac{2\tilde{P}_k}{1.1}$$

	$n_t = 4$	$n_t = 8$	$n_t = 16$	$n_t = 32$
PFGD	0.0018	0.0021	0.0026	0.0048
SeDuMi	0.3245	0.3537	0.5236	0.9415
MOSEK	0.1299	0.1404	0.1805	0.3335

Summary

- MIMO capacity under multiple simultaneous power constraints
  - Analytical solution: MISO and some special cases of MIMO
  - PFGD algorithm for general MIMO
  - PFGD algorithm for general MIMO with rank constraints

- Lower complexity that standard solvers
  - Structure of the problem is used to simplify
  - Directly solves for precoding matrix
  - Accurate solution in simulation study, local convergence result

Thank you

http://www.ee.iitm.ac.in/~skrishna/

R. Chaluvadi, S. S. Nair, S. Bhashyam, Optimal Multi-antenna Transmission with Multiple Power Constraints, Submitted to IEEE Transactions on Wireless Communications, 2018.
S. S. Nair, R. Chaluvadi, S. Bhashyam, Optimal Rank-constrained Transmission for MIMO under Per-group Power Constraints, Proceedings of WCNC 2017, San Francisco, CA, USA, March 2017.
R. Chaluvadi, S. S. Nair, S. Bhashyam, Optimal Multi-antenna Transmission with Per-group and Joint Power Constraints, Proceedings of WCNC 2017, San Francisco, CA, USA, March 2017.